Theory of Ion Acceleration with Closed Electron Drift

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A theory is developed for $E \times H$ acceleration of ions in channels with closed electron drift paths. Two classes of solutions were found depending on whether the electron temperature was constant throughout the acceleration channel or allowed to rise as the anode was approached. In the case of constant electron temperature a continuous potential variation was obtained. This solution corresponds, from the experimental viewpoint, to the Hall-current accelerator. If the electron temperature was allowed to rise, a near discontinuous jump was predicted at the positive end of the acceleration channel. This solution corresponds to the anode-layer accelerator described in Soviet literature.

Nomenclature

B = magnetic induction, T

e = absolute electronic charge, 1.6022×10^{-19} C

E = electric field, V/m

H = magnetic intensity, A-turns/m

 j_e = electron current density, A/m²

 m_e = electron mass, 9.1095×10^{-31} kg

 $n = \text{electron density, m}^{-3}$

 n_0 = electron density after acceleration, m⁻³

T = electron temperature, eV

 v_x = axial electron velocity, m/s

 v_{θ} = circumferential electron velocity, m/s V = electric potential, relative to exhaust-

 \dot{V} = electric potential, relative to exhaust-plane potential,

 V_0 = electric potential difference through which ions are accelerated, V

 axial distance in direction of electron diffusion, measured from exhaust plane, m

Introduction

CLASS of ion accelerators, called Hall-current accelerators in the United States, use the electric field generated by electrons diffusing through a magnetic field to accelerate ions. To avoid large electron sources and sinks due to $E \times H$ drift, the acceleration channels have closed drift paths usually annular in shape. A theory is developed herein for the ion acceleration with closed electron drift paths.

A sketch of a closed-drift thruster is shown in Fig. 1. Some of the electrons from the cathode neutralize the accelerated ion beam, while other electrons flow upstream to the anode. The electrons flowing upstream have two components of motion. The circumferential motion (v_{θ}) in Fig. 1) follows a closed path and is the Hall current. The axial component (v_x) in Fig. 1) is the motion analyzed herein.

The major assumptions in the theory developed herein are:
1) the radial and circumferential variations in properties are small so that the time-averaged properties vary only with the coordinate x, which is aligned with the acceleration direction;
2) the ion current density is constant throughout the acceleration region; and 3) the electron current density is

constant throughout this region. Assumptions 2) and 3) imply (with energy conservation for ions) a constant-area flow channel and localized ion generation at the upstream end of the acceleration channel. In practice, constant ion current density usually means a channel of uniform radius and length as well. Using an integral approach to electron diffusion that was developed previously, it is not necessary to make assumptions regarding the detailed distribution of magnetic field intensity.

Differential Diffusion Equation

Assuming Bohm diffusion^{2,3} for both the potential and density gradient terms, the equation governing electron diffusion in the upstream direction is

$$j_e = \frac{en}{16B} \frac{\mathrm{d}V}{\mathrm{d}x} - \frac{eT}{16B} \frac{\mathrm{d}x}{\mathrm{d}n} \tag{1}$$

Note that this equation is concerned only with the electron motion opposite to the applied electric field, and not the circulating or Hall current.

If ion production and losses are both negligible in the acceleration region, and the ion acceleration is restricted to the x direction, then the ion density can be expressed as

$$n = n_0 V_0^{1/2} / (V_0 - V)^{1/2}$$
 (2)

From the quasineutrality assumption for a plasma, Eq. (2) also gives the electron density. From Eq. (2), one might expect an infinite electron and ion density at V_0 , which is the origin potential for the ions. In practice, the ions actually originate over a small range of potential, thus avoiding the singularity of infinite density. Because the concern herein is with potential differences that are large compared to this small range of origin potential, Eq. (2) is an accurate approximation.

Zero Electron Temperature

The simplest case to calculate is the one in which electron temperature is negligible throughout the acceleration region. This does not actually mean a zero electron temperature, but simply that the electron temperature (in eV) is far smaller than the total acceleration potential difference V_{θ} (in V) throughout. Substituting Eq. (2) in Eq. (1) and rearranging the results,

(3)

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$$16Bj_e dx = en_0 V_0^{1/2} dV / (V_0 - V)^{1/2}$$

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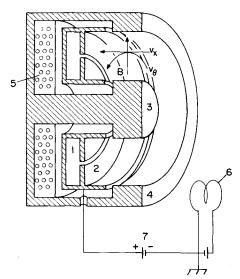


Fig. 1 Hall-current or closed-drift thruster. (1) Propellant distributor; (2) anode; (3) and (4) pole pieces; (5) magnet winding; (6) cathode; and (7) discharge power supply.

With j_e assumed a constant throughout the acceleration region, Eq. (3) can be integrated from the exhaust plane (V=0) to obtain[†]

$$16j_e \int B dx = e n_0 V_0 \left[2 - 2 \left(1 - V / V_0 \right)^{1/2} \right]$$
 (4)

To facilitate comparison with other solutions, it is convenient to rearrange the terms

$$j_e \int B dx / e n_0 V_0 = [1 - (1 - V/V_0)^{1/2}] / 8$$
 (5)

For integration over the entire acceleration region to $V = V_0$, the right-hand side of Eq. (5) becomes $\frac{1}{2}$ 8.

In practice, the acceleration process described by Eq. (5) might result from the continual collection of high-energy electrons by, and emission of low-energy electrons from, the walls of the acceleration channel. The electron temperature might also be maintained at a low value by excitations of ions and neutrals. In the latter case, however, one might also expect the assumption of negligible ionization in the acceleration region to be violated.

Regardless of the mechanism of electron exchange that is assumed at the acceleration channel walls, the high electrical conductivity of the plasma parallel to the radial magnetic field direction will result in roughly constant electrical potential along each field line. Thus, the acceleration of the ions will be substantially in the axial direction, as required to provide thrust.

Zero Initial Electron Temperatures

As another fairly simple case, the electron temperature can again be assumed zero at the exhaust plane, but then to increase as the electrons pass through the acceleration region. If energy is conserved, one would expect the electron temperature to be

$$T = 2V/3 \tag{6}$$

That is, the energy gained in passing through a potential difference of V results in a rms electron temperature of 2V/3.

Note that the "zero" electron temperature is again simply assumed to be small compared to V_0 . Typical values of V_0 are several hundred volts so that the initial temperature can be several electron volts and still be negligible compared to V_0 .

The electron temperature variation of Eq. (6) assumes that thermal conduction by the electron population is negligible in the x direction. The assumed Bohm diffusion is believed to result from electron scattering by ion-plasma waves, which can be generated by much lower electron-ion drift velocities than electron-plasma waves. This electron scattering is usually orders of magnitude greater than the scattering due to "soft" or Coulomb collisions by other electrons. Yet the thermal conductivity of an electron population transverse to a magnetic field is primarily a function of these Coulomb collisions. The scattering collisons with ion-plasma waves tend to result in a much slower transfer of energy to and from the electron population. Ignoring thermal conduction by electrons in the x direction thus appears to be consistent with the assumption of Bohm diffusion.

The thermal conductivity of the plasma parallel to the magnetic field (the radial direction in Fig. 1) is, of course, quite high. For the conservation of energy assumed in Eq. (6), the radial escape of electrons must be prevented. Experimentally, this escape is prevented by sheaths at the inner and outer radii of the acceleration channel. For these sheaths to develop the inner and outer channel walls should not be significant electron sources, either from secondary electron emission or any other cause.

The sheath thicknesses, due to the high plasma densities, will be small compared to the width of the acceleration channel. The acceleration of the bulk of the ions thus will be unaffected by the presence of the sheaths.

For a nonzero electron temperature, the second term of Eq. (1) becomes important. The expression for n, Eq. (2), can be differentiated to obtain

$$dn = n_0 V_0^{1/2} dV / 2 (V_0 - V)^{3/2}$$
(7)

Substituting Eqs. (2), (6), and (7) into Eq. (1), and rearranging the results,

$$16Bj_{e}dx = en_{0}V_{0}^{1/2} \left[\frac{-dV}{(V_{0} - V)^{1/2}} + \frac{VdV}{3(V_{0} - V)^{3/2}} \right]$$
 (8)

Again assuming j_e a constant throughout the acceleration region, Eq. (8) can be integrated from the exhaust plane (V=0) to obtain

$$j_e [Bdx/en_0 V = \frac{1}{24} [5 - 4(1 - V/V_0)^{\frac{1}{12}} - 1/(1 - V/V_1)^{\frac{1}{12}}]$$
 (9)

This result is not as straightforward as Eq. (5). For example, integration of V to the limit of V_0 results in an infinitely negative value of the right-hand side of Eq. (9). To illustrate the problem in more detail, Eq. (9) is plotted in Fig. 2 together with Eq. (5) for comparison.

The values in Fig. 2 represent integrals from V=0 to a particular value of V/V_0 . For the integration limit of V/V_0 much closer to zero than unity, the two approaches [Eqs. (5) and (9)] give very similar results. But only Eq. (9) has a diffusion in the opposite direction due to the increase in plasma density as potential increases. Note that the assumption of zero (negligible) electron temperature for Eq. (5) results in no such opposing diffusion. Due to this opposing diffusion effect in Eq. (9), less additional magnetic field is required to offset an additional potential difference at the same current density. The effect is proportional to electron temperature, therefore, the difference between the two solutions increases with V/V_0 . [Note that from Eq. (6), T/V_0 is proportional to V/V_0 .]

[†]The integral of $\int Bdx$ is assumed to have B normal to dx for the problem herein. If this orientation is not used; a cross product must be used in the integration, i.e., $\int B \times dx 1$.

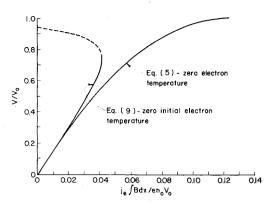


Fig. 2 Comparison of theoretical solutions for acceleration region in steady-state plasma thruster.

At a $V/V_0 = \frac{3}{4}$, the opposing diffusion due to increasing plasma density exactly balances the diffusion due to potential gradient. An incremental increase in V/V_0 at this point thus requires no additional magnetic field to maintain a given value of current density. Beyond $V/V_0 = \frac{3}{4}$, the solutions become double-valued, which is clearly unrealistic.

In the derivations of Eqs. (5) and (9), the diffusion processes were assumed to be continuous. That is, the finite sizes of electron orbits were not considered. The physical expectation for Eq. (9), then, is that the electrons would pass from $V/V_0 = \frac{3}{4}$ to $V/V_0 = 1$ by means of a single, collision-free orbit. Using momentum considerations, the magnetic-field integral that can be crossed by this single orbit is

$$\int B dx = (2m_o/e)^{\frac{1}{2}} (2T^{\frac{1}{2}} + V_o^{\frac{1}{2}}/2)$$
 (10)

where T is the electron temperature (in eV) at the start of the orbit and the potential difference from beginning to end of the orbit is $V_0/4$.‡ The rms velocity was used to calculate the contribution in Eq. (10) due to random electron energy. Having passed through a potential difference of $3V_0/4$, Eq. (6) indicates an electron temperature of $V_0/2$ at the start of the orbit for substitution in Eq. (10). With this substitution, together with the electron charge and mass, the magnetic integral crossed is

$$\int B dx = 6.45 \times 10^{-6} V_0^{1/2}$$
 (11)

The maximum value of abscissa in Fig. 2 (the start of the escape orbit) is

$$j_e \int B dx / e n_0 V_0 = 0.0417$$
 (12)

Substituting for electronic charge and solving for the magnetic integral,

$$\int B dx = 6.68 \times 10^{-21} n_0 V_0 / j_0$$
 (13)

It should be clear from Eqs. (11) and (13) that the escape orbit does not correspond to any particular fraction of the diffusion magnetic integral indicated in Fig. 2. Therefore, it is not possible to show the specific value of abscissa in Fig. 2 that is associated with the end of the escape orbit.

The escape orbit region can also be described as a sheath. Due to the presence of the magnetic field this sheath is reversed in potential from the usual electrostatic sheath formed in the absence of a magnetic field. If the ion

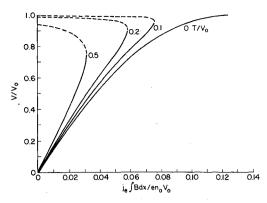


Fig. 3 Solutions for constant electron temperatures. $(T/V_{\theta} \text{ in eV/V.})$

generation is at an electrode surface, such as by a contact ionization surface, then this is the only sheath present at the upstream end of the acceleration region. If the ion generation takes place in a plasma, as is more frequently the case, then there will be additional plasma sheath effects that should be included. These additional effects will be discussed in the following sections.

Constant Electron Temperature

The constant electron temperature case is the more general formulation of the zero electron temperature case. For a constant electron temperature, Eqs. (2) and (7) are substituted into Eq. (1),

$$16Bj_{e}dx = en_{0}V_{0}^{1/2} \left[\frac{-dV}{(V_{0} - V)^{1/2}} + \frac{TdV}{2(V_{0} - V)^{3/2}} \right]$$
(14)

Integration upstream from the exhaust plane, again assuming a constant j_e , yields

 $j_e \int B dx / e n_0 V_0$

$$= \frac{1}{8} \left[1 + \frac{T}{2V_0} - (1 - V/V_0)^{\frac{1}{2}} - \frac{T/V_0}{2(1 - V/V_0)^{\frac{1}{2}}} \right]$$
 (15)

The numerical results for Eq. (15) are similar to those for Eq. (9), in that results become double valued for V/V_0 above a certain value. For Eq. (15), the limit for single-valued operation is

$$V/V_0 = I - T/2V_0 \tag{16}$$

Values for Eq. (5) $(T/V_0 = 0)$ and Eq. (15) are shown in Table 1 and Fig. 3.

At first glance, the results shown in Table 1 and Fig. 3 indicate that a potential jump may be expected for acceleration with constant electron temperature, similar to that shown in Fig. 2 for zero initial electron temperature. If the ions are assumed to come from a plasma at the start of acceleration, they should have an initial velocity corresponding to ion acoustic velocity (the minimum velocity found by Bohm for a stable plasma sheath). Assuming the same fixed temperature for Maxwellian electrons in this plasma, the preacceleration to ion acoustic velocity will be just enough to avoid the dashed portions of the curves shown in Fig. 3, hence avoid any sudden potential jump.

For example, a constant electron temperature of $0.1V_0$ would result in the preacceleration to Bohm velocity, through a potential difference of $0.05V_0$. This preacceleration within the ion production plasma would be just sufficient to avoid the dashed portion of the curve for $T/V_0 = 0.1$ in Fig. 3.

[‡]Equation (10) can be obtained by adding the magnetic integral contributions of Eqs. (19) and (20) of Ref. 1.

| Table 1 | Solutions for constant electron tem | peratures [Eqs. (5) | and (15). Valu | es of $i_a \langle Bdx/en_a V_a \rangle$ |
|---------|-------------------------------------|---------------------|----------------|--|
| | | | | |

| $\frac{V}{V_0}$ | $\frac{T}{V_0} = 0$ | $\frac{T}{V_0} = 0.1$ | $\frac{T}{V_0} = 0.2$ | $\frac{T}{V_0} = 0.5$ |
|-----------------|---------------------|-----------------------|-----------------------|-----------------------|
| 0.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.1 | 0.0064 | 0.0061 | 0.0057 | 0.0047 |
| 0.2 | 0.0132 | 0.0125 | 0.0117 | 0.0095 |
| 0.3 | 0.0204 | 0.0192 | 0.0180 | 0.0143 |
| 0.4 | 0.0282 | 0.0264 | 0.0245 | 0.0191 |
| 0.5 | 0.0366 | 0.0340 | 0.0314 | 0.0237 |
| 0.6 | 0.0459 | 0.0423 | 0.0387 | 0.0278 |
| 0.7 | 0.0565 | 0.0514 | 0.0462 | 0.0307 |
| 0.8 | 0.0391 | 0.0614 | 0.0536 | 0.0305^{a} |
| 0.9 | 0.0855 | 0.0720 | 0.0584 | 0.0179 ^a |
| 0.9375 | 0.0938 | 0.0750 | 0.0562a | 0.0000^{a} |
| 0.95 | 0.0970 | 0.0753 | 0.0536 ^a | -0.0115^{a} |
| 0.9975 | 0.1188 | 0.0000^{a} | 0.1188 ^a | -0.3562^{a} |
| 1.0 | 0.1250 | - ∞ ^a | - ∞ ^a | ∞ ^a |

^aThese values are believed to be in a physically unrealistic operating regime.

Table 2 Solutions for conserved electron energy [Eqs. (9) and (19). Values of j_e [Bdx/en $_\theta V_\theta$]

| $\frac{V}{V_0}$ | $\frac{T}{V_{\theta}} = 0.$ | $\frac{T}{V_0} = 0.1$ | $\frac{T}{V_0} = 0.2$ | $\frac{T_0}{V_0} = 0.5$ |
|-----------------|-----------------------------|-----------------------|-----------------------|-------------------------|
| 0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.1 | 0.0063 | 0.0060 | 0.0056 | 0.0046 |
| 0.2 | 0.0127 | 0.0119 | 0.0112 | 0.0090 |
| 0.3 | 0.0191 | 0.0179 | 0.0166 | 0.0130 |
| 0.4 | 0.0254 | 0.0236 | 0.0218 | 0.0163 |
| 0.5 | 0.0316 | 0.0290 | 0.0264 | 0.0186 |
| 0.5625 | 0.0351 | 0.0319 | 0.0287 | 0.0191 |
| 0.6 | 0.0370 | 0.0334 | 0.0298 | 0.0189 ^a |
| 0.675 | 0.0402 | 0.0355 | 0.0308 | 0.0167 ^a |
| 0.7 | 0.0410 | 0.0358 | 0.0307 ^a | 0.0152 ^a |
| 0.7125 | 0.0413 | 0.0359 | 0.0304^{a} | 0.0142^{a} |
| 0.75 | 0.0417 | 0.0354 ^a | 0.0296 ^a | 0.0104^{a} |
| 0.8 | 0.0406a | 0.0329^{a} | 0.0252a | 0.0020^{a} |
| 0.8086 | 0.0402 ^a | 0.0321 ^a | 0.0241a | 0.0000^{a} |
| 0.8944 | 0.0260a | 0.0130^{a} | 0.0000^{a} | -0.0389^{a} |
| 0.9 | 0.0239a | 0.0104^{a} | -0.0032^{a} | -0.0437^{a} |
| 0.9173 | 0.0175 ^a | 0.0000^{a} | -0.0155^{a} | -0.0620^{a} |
| 0.9375 | 0.0000^{a} | -0.0188^{a} | -0.0375^{a} | -0.0938^{a} |
| 1.0000 | _ ∞ ^a | - ∞ ^a | - ∞ ^a | - ∞ ^a |

^aThese values are believed to be in a physically unrealistic operating regime.

In applications in which the ions are generated in a plasma, a constant electron temperature therefore can be expected to result in a solution with a continuous potential variation throughout the acceleration region.

Conserved Electron Energy

The conserved electron energy case is the more general formulation of the zero initial initial electron temperature case. The electron energy was assumed to be conserved in the derivation of Eq. (9), but the initial electron temperature in the exhaust plane, T_0 , was assumed to be zero. The case of interest here is a nonzero initial temperature. The electron temperature as a function of local potential V then becomes

$$T = T_0 + 2V/3 (17)$$

Substituting Eqs. (2), (7), and (17) into Eq. (1) yields

 $16j_e B dx/en_0 V_0^{\frac{1}{2}}$

$$= \frac{\mathrm{d}V}{(V_0 - V)^{\frac{1}{2}}} - \frac{T_0 \mathrm{d}V}{2(V_0 - V)^{\frac{3}{2}}} - \frac{V \mathrm{d}V}{3(V_0 - V)^{\frac{3}{2}}}$$
(18)

Integrating from the exhaust plane then gives

 $j_e \int B dx / e n_0 V_0$

$$= \frac{1}{24} \left[5 + \frac{3T_0}{2V_0} - 4(1 - V/V_0)^{\frac{1}{2}} - \frac{2 + 3T_0/V_0}{2(1 - V/V_0)^{\frac{1}{2}}} \right]$$
(19)

The limit for single-valued results is

$$V/V_0 = (3/4)(1 - T_0/2V_0)$$
 (20)

Values for Eq. (9) $(T_{\theta}/V_{\theta}=0)$ and Eq. (19) are shown in Table 2 and Fig. 4.

A finite initial electron temperature, T_0 , is shown to result in the requirement for a potential jump, similar to that described in connection with Eq. (9) and Fig. 2. It is true that the magnitude of this jump decreases relative to V_0 as T_0 increases, when an initial ion velocity equal to the ion acoustic velocity is assumed. But the jump will still exist for all $T_0/V_0 < 1$.

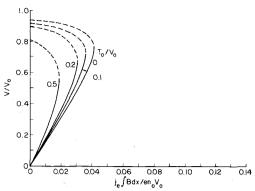


Fig. 4 Solutions for conserved electron energy and various exhaustplane electron temperatures. $(T_{\theta}/V_{\theta} \text{ in eV/V.})$

Comparison with Experimental Results

High performance of a closed-drift accelerator is associated with low values of the electron current back through the acceleration channel. At the microscopic level, this electron backflow is determined by the diffusion coefficient. The diffusion coefficient used herein is attributed to Bohm, with the numerical constant equal to the most widely used value.^{2,3}

$$D_B = T_e/16B \tag{21}$$

Although this value has been used to correlate diffusion data for a wide range of experiments,³ it should be kept in mind that it is only a semiempirical approximation and there is considerable data scatter. Perhaps more important, anomalous diffusion has been found to follow 1/B relationships in a wide variety of tests. Further, if one accepts a 1/B relationship proportional to Eq. (21), the results of Eqs. (15) and (19) should be expected to follow as qualitative consequences.

The constant electron temperature solution, Eq. (15), does not depart significantly from our intuitive expectations. The solution with conserved electron energy, Eq. (19), is less obvious. A similar sudden potential jump at the positive end of the accelerating region, though, has been described in Soviet literature.⁵ The continous potential variation of Eq. (15) is associated with a "hybrid accelerator with closedelectron drift," which is normally translated as a "Hallcurrent accelerator." The version where the potential jump is encountered is called an "accelerator with anode layer," with no equivalent type in U.S. literature. Both of these accelerator types are included in the overall classification of "accelerators with closed (electron) drift."

An experimental study was made in which both accelerator types were studied with essentially the same experimental apparatus. The only essential difference was that insulators were included for the Hall-current accelerator and omitted for the anode-layer version. In Fig. 1, these insulators would be located between the anode and pole piece 3 and between the anode and pole piece 4, with the exposed surfaces of both flush with the inner and outer walls of the acceleration channel. The presence of the insulator in close proximity to the accelerating channel would be expected to result in the collection of the higher energy electrons, and their replacement with lower energy secondaries. The presence of the insulator thus would be expected to correspond more to Eq. (15). Without the insulator, there would be a barrier of lower density plasma between the plasma in the accelerating channel and any solid surface. The electrons flowing back from the neutralizer would thus be expected to reach a higher temperature before escaping, and the acceleration process would be expected to correspond more to Eq. (19).

Since the initial conference presentation of this paper, the theory contained herein has been compared to experimental results from a number of investigations.8 For configurations

with a high azimuthal uniformity, low ion losses to channel walls, and moderate values of overall magnetic integral, the experimental electron currents were within about a factor of 3 of the theoretical values predicted herein. For at least some of the experimental data, optimization was probably also involved. As described by Ivaschenko et al., 9 small differences in operating conditions can lead to large differences in performance.

Conclusions

Solutions were derived for ion accelerators with closed electron drift. Two classes of solutions were found, depending on whether the electron temperature was constant throughout the acceleration channel or allowed to rise as electrons flow toward the anode. In the latter case, a near discontinous jump in potential was predicted at the positive end of the channel. Details of the acceleration process (such as the initial electron temperature) do change particulars of the calculated results. But the existence of the potential jump appears to be a persistent phenomenon when electron potential energy is convered into random energy.

The continous potential solution associated with the assumption of constant electron temperature corresponds, from the experimental viewpoint, to the Hall-current accelerator. The solution with a potential jump near the anode corresponds to the anode-layer accelerator, described in Soviet literature.

A prior analysis of closed-drift acceleration was carried out assuming $1/B^2$ electron diffusion instead of the more realistic 1/B diffusion used herein. 10 The author was not aware of this prior analysis at the time the original conference version of this paper was presented, but is is apparently the initial reference for Soviet anode-layer thrusters. With the use of $1/B^2$ diffusion, the results of this prior analysis do not show agreement with experimental results. The essential feature of a near discontinous potential jump near the anode is described, however, using a logic roughly similar to that presented herein.

Acknowledgment

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